Figure Supplements

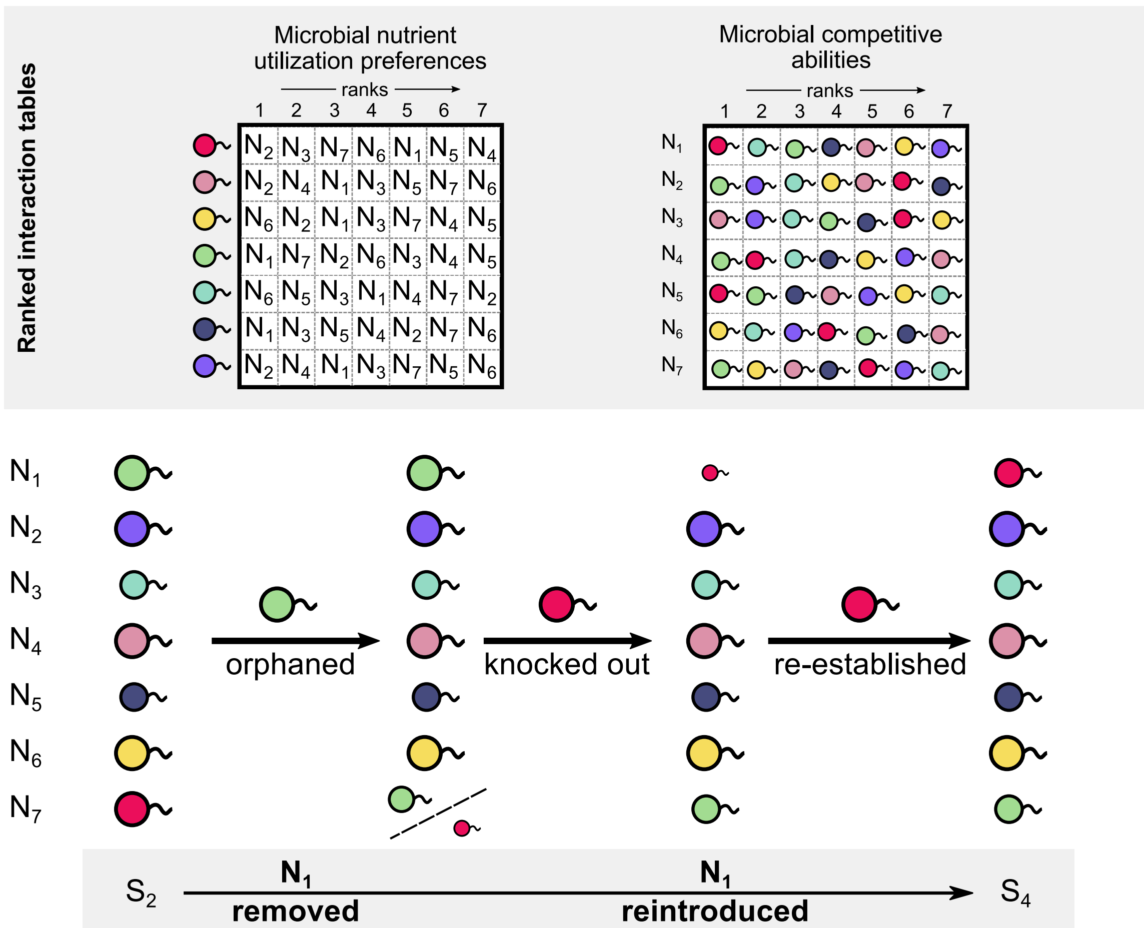

Figure S1: Specific steps during community transitions from one stable state to another. A detailed step-by-step breakdown of how the microbial community in our example in figure **??** of the main text can transition from one stable state (here S_2) to another (here S_4) via a very specific perturbation: the removal of nutrient N_1 and its reintroduction at the specific time-point shown thereafter. First, the green microbe is left without its preferred growth nutrient (N_1). It then attempts to compete for its next preferred nutrient, N_7, competitively displaces the red microbe, which can then re-establish on N_1 reintroduced at that specific time. The resultant community now exhibits the alternate stable state, S_4.

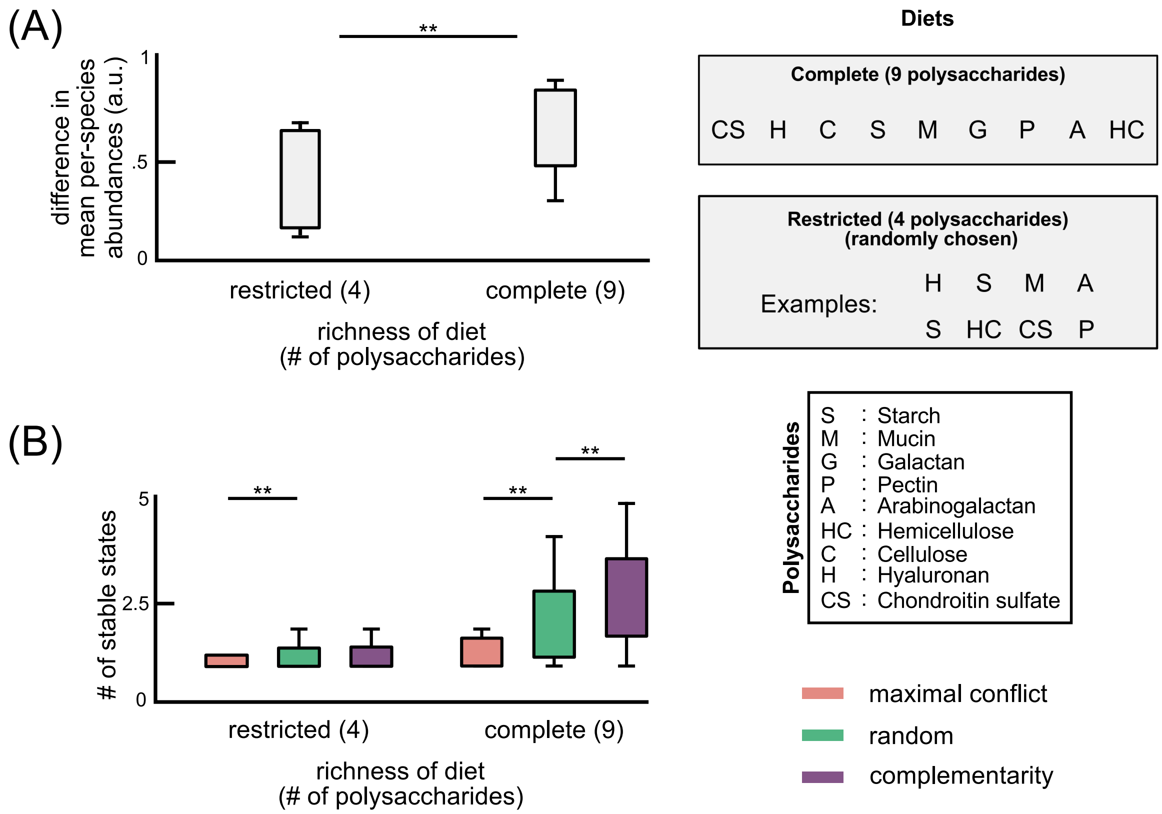

Figure S2: Contrast between restricted and complete diets in *Bacteroides* species.
(A) In figure **??** in the main text, we show that using different nutrient preferences (complementary, random and maximally conflicting) for a realistic community of *Bacteroides* species can result in different species abundance profiles. Specifically, complementary lists lead to higher abundances for all *Bacteroides* species, whereas conflicting lists result in low abundances. However, we showed this assuming a complete 'diet' with all 9 consumable polysaccharides available. Here, we show that the difference between communities with complementary and conflicting preferences (in our model) shrinks when the diet is 'restricted', i.e. when only about half the polysaccharides are available, and randomly selected. This is consistent with an increased expectation for complementary nutrient preferences between co-occurring microbes in environments with richer diets. **(B)** The number of stable states, as described in the main text (see Materials and methods: Enumerating all stable states) for all three cases of microbial nutrient preferences for restricted and complete diets. Complete diets typically have a higher number of stable states (typically ~ 2) for complementary preferences than either random or conflicting preferences. In some cases, the number of stable states is higher (i.e. $4-5$), and these cases are more likely if the preferences are complementary. (In all cases, we use the Kolmogorov-Smirnov test to compare distributions, with a P-value threshold of 0.01.)

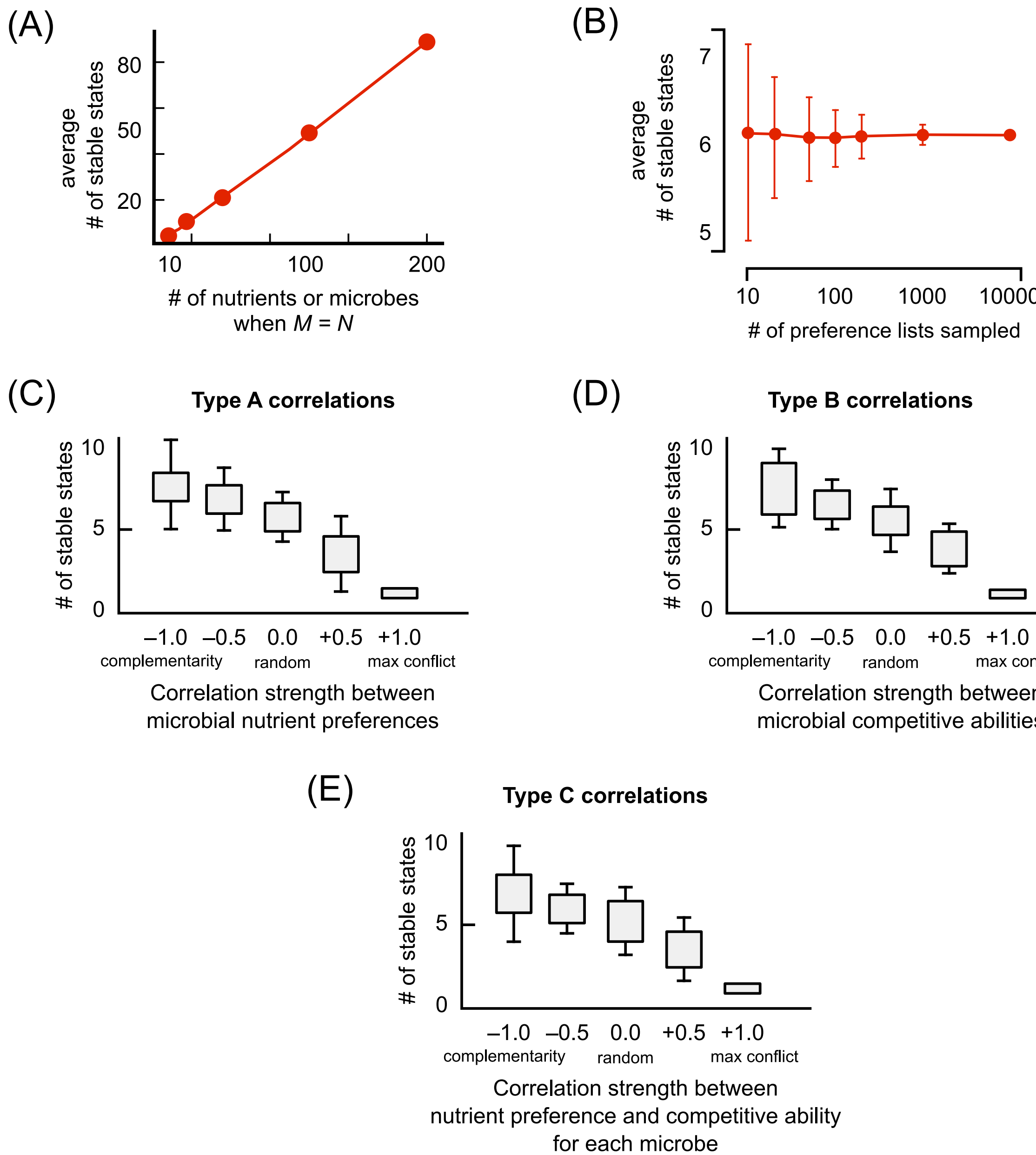

Figure S3: Characteristics of the number of stable states according to our model
(A) The average number of stable states for $1,000$ randomized preference lists (when $M = N$ on the x-axis, M and N being the number of microbes and nutrients respectively). **(B)** For $M = N = 10$, this plot shows the average number of stable states using randomized preference lists of different sizes (on the x-axis). The error-bars represent the standard deviation among different samples of the same size. **(C, D, E)** Box plots of the number of stable states for different type A (microbes' nutrient preferences), type B (microbes' competitive abilities for a nutrient) and type C (preference for a nutrient and competitive ability on it for a particular microbe) correlations (here $M = N = 10$). Correlations strengths are shown on the x-axis. They can either be complementary (anti-correlated preferences, or strength -1), random (uncorrelated preferences, or strength 0), or maximally conflicting (strongly positively correlated preferences, or strength $+1$). For all types of correlations, strong positive correlations result in a community with only one unique stable state.

Supplementary Methods

Studying correlations between preference lists

We assumed a microbial community with 10 microbes and 10 nutrients (i.e., $M = N = 10$). For each community, we generated a unique set of preferences that were correlated according to a chosen strength x. For each microbial species i on each nutrient α, we first generated two numbers: one indicating the microbes preference for that nutrient, and the other for its competitive ability to uptake that nutrient given competition with others. In other words, we generated "preference values" $P_{i\alpha}$ and "competitive ability values" $C_{i\alpha}$ for each microbe i on each nutrient α such that these values reflected our desired correlations of strength x.

After generating these values, we rank ordered the nutrient preferences and competitive abilities to generate the two ranked tables that fully specify our stable marriage model, and used our previously described algorithm to enumerate the number of stable states (see Methods in the main text). Below we describe our method to generate three types of correlations (see Discussion).

First, type A correlations indicate correlations between the nutrient preferences of all microbes, i.e. they reflect a case when certain nutrients are "universally prized" or preferred. Here, we randomized all competitive ability values $C_{i\alpha}$'s from a uniform distribution between 0 and 1. For nutrient preferences, we assigned $P_{i\alpha}$'s in the following way.

$$P_{i\alpha} = x\lambda\alpha + \sqrt{1-x^2}\lambda'_{i\alpha} \qquad \forall \qquad 0 \leq x \leq 1. \tag{1}$$

All λ's represent random numbers distributed uniformly between 0 and 1. Note that for each i and α, we generate a unique λ', but a shared λ. Further, here $0 \leq x \leq 1$ represents positively correlated (conflicting) preferences. For $1 \leq x < 0$ which represents negatively correlated (complementary) preferences, we used our original procedure to generate complementary nutrient preferences (see Methods in the main text), with the following change. At $x = -1$ (maximum complementarity), we always assigned a unique nutrient as a microbe's top choice, but at $x = -0.5$, we assigned a unique nutrient with probability $|x| = 0.5$.

Second, type B correlations indicate correlations between the competitive abilities of all microbes on the same nutrient. Here, we randomized microbes nutrient preferences $P_{i\alpha}$'s and used a similar algorithm for the type A correlations above, except with competitive abilities $C_{i\alpha}$'s, as follows.

$$C_{i\alpha} = x\lambda\alpha + \sqrt{1-x^2}\lambda'_{i\alpha} \qquad \forall \qquad 0 \leq x \leq 1. \tag{2}$$

Finally, type C correlations indicate correlations between a microbe's preference for a nutrient and its competitive ability on that nutrient. For this, for each microbe i and nutrient α, we generate both the preference and competitive ability values using the following equations.

$$P_{i\alpha} = \lambda_{i\alpha} + \sqrt{1-x^2}\lambda'_{i\alpha} \tag{3}$$

$$C_{i\alpha} = x\lambda_{i\alpha} + \sqrt{1-x^2}\lambda''_{i\alpha}. \tag{4}$$